Logic of a proof...

How do we solve this puzzle?

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What are we going to learn in this lesson?

- What is a proof?
- How to structure a two-column proof?

What is a proof?

"The ideal reasoner would, when he has once been shown a single fact in all its bearing, deduce from it not only all the chain of events which led up to it, but also all the results which would follow from it."

-Sherlock Holmes, The Five Orange Pips

A proof is a set of statements and reasons. The following can be used as reasons in a proof...
- given information
- definitions
- properties of real numbers
- previously proved theorems
- postulates or axioms

A theorem: statement that has already been proved.

A postulates/axioms: statement that is accepted to be true without proof.
2a geometric proofs

The structure of a proof

One way to record the statements and reasons for a proof is to use two columns, with **statements** on the left and **reasons** on the right. This method is called a two-column proof.

The structure of a proof follows these steps...

1. Draw a diagram and mark the given information on the diagram.
2. Analyze the given information and the diagram.
3. Analyze what is to be proved.
4. Use the analyses to plan a logical progression from the given information to what is to be proved.

Example

1. Complete the following proof

Given: $AE = CE; ED = EB$

Prove: $AD \parallel BC$

**Statement** | **Reasons**
---|---

a) $AE = CE$

b) $ED = EB$

c) $\angle AED = \angle CEB$

d) $\triangle AED \cong \triangle CEB$

e) $\angle DAE = \angle BCE$

f) $AD \parallel BC$
2a geometric proofs

Example:
2. Given: \( BA = BC; BD = BE \)
Prove: \( \angle ABD = \angle CBE \)

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Practice
1. Given: \( \angle PRT = \angle PSQ; PR = PS \)
Prove: \( RT = SQ \)

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2a geometric proofs

Practice
2. Given: $AB = CD; AD = CB$
Prove: $AD \parallel BC$

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Worksheet
p. 389-390
#s 5-10
Exit slip

Given: HJ is the perpendicular bisector of KI

Prove: HK = HI

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\[ \begin{align*}
AD &= AE \\
DB &= EC \\
AB &= AC \\
\angle BAE &= \angle CAD \\
\triangle BAE &\cong \triangle CAD \\
BE &= CD
\end{align*} \]

- Given
- Given
- Sum of equal corresponding sides.
- Common
- SAS
- Two corresponding sides of congruent triangles.
Given

\[ CA = CE \]  
\[ \angle A = \angle CDB \]  
\[ \angle E = \angle CBD \]  
\[ \triangle CAE \text{ is isosceles} \]  
\[ \angle A = \angle E \]  
\[ \angle CDB = \angle CBD \]  
\[ \triangle CBD \text{ is isosceles} \]

---

**Isosceles Triangle Theorem**

Given

\[ A : B \]  
\[ B : C \]  
\[ A : C \]
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Given

\[ QR = QT \]
\[ SR = ST \]
\[ QS = QS \]
\[ \triangle QRS \cong \triangle QTS \]
\[ \angle QRS = \angle QTS \]

Given

Given

Common

SSS

Two corresponding angles of congruent triangles.
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$\angle 1 = \angle 2$  
$\angle 3 = \angle 4$  
$AB = AE$  
$\triangle ABC$ is isosceles  
$\triangle AED$ is isosceles  
$AB = AC$  
$AD = AE$  
$AC = AD$  
$\triangle ACD$ is isosceles

Given  
Given  
Given  
ITT  
ITT  
ITT (Converse)  
ITT (Converse)  
Transitivity  
ITT
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\[ DE = DG \quad \text{Given} \]
\[ FE = FG \quad \text{Given} \]
\[ DF = DF \quad \text{Common} \]
\[ \triangle DEF \cong \triangle DGF \quad \text{SSS} \]
\[ \angle EFH = \angle GFH \quad \text{Two corresponding angles of congruent triangles} \]
\[ FH = FH \quad \text{Common} \]
\[ \triangle FGH \cong \triangle FHE \quad \text{SAS} \]
\[ EH = GH \quad \text{Two corresponding sides of congruent triangles} \]
Given: \( LM \perp JK \quad KN \perp JN \quad MK = NL \quad KL = KL \)

Perpendicular definition

HL

Two corresponding angles of congruent triangles

ITT

\( \triangle JKL \) est isocèle